Observation of Dispersive Shock Waves, Solitons, and Their Interactions in Viscous Fluid Conduits

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Dispersive shock waves and solitons are fundamental nonlinear excitations in dispersive media, but dispersive shock wave studies to date have been severely constrained. Here, we report on a novel dispersive hydrodynamic test bed: the effectively frictionless dynamics of interfacial waves between two high viscosity contrast, miscible, low Reynolds number Stokes fluids. This scenario is realized by injecting from below a lighter, viscous fluid into a column filled with high viscosity fluid. The injected fluid forms a deformable pipe whose diameter is proportional to the injection rate, enabling precise control over the generation of symmetric interfacial waves. Buoyancy drives nonlinear interfacial self-steepening, while normal stresses give rise to the dispersion of interfacial waves. Extremely slow mass diffusion and mass conservation imply that the interfacial waves are effectively dissipationless. This enables high fidelity observations of large amplitude

from the full set of coupled Navier-Stokes fluid equations using a perturbative procedure with the viscosity ratio as the small parameter [30]. The conduit equation (1) was theoretically shown to be valid for long times and large amplitudes under modest physical assumptions on the basin geometry, background velocities, fluid compositions, weak mass to momentum diffusion, and characteristic aspect ratio. The efficacy of this model has been experimentally verified in the case of solitons [29,31].

The study of DSWs involves a nonlinear wave modulation theory, commonly referred to as Whitham theory [22], which treats a DSW as an adiabatically modulated periodic wave [1,23]. Using Whitham theory and Eq. (1), key conduit DSW physical features such as the leading soliton amplitude and leading or trailing speeds have been determined [24]. For the jump in downstream to upstream area ratio $\_$, Whitham theory applied to the conduit equation (1) predicts relatively simple expressions for the DSW leading $_{+}$ and trailing $_{-}$ edge speeds,

$$_{+} = \sqrt{1 + 8\,\_} - 1, \qquad _{-} = 3 + 3\,\_ - 3\sqrt{\_(8 + \_)}, \qquad (2)$$

in units of the characteristic speed $_0 = L_0/T_0$. The leading edge approximately corresponds to an isolated soliton where the modulated periodic wave exhibits a zero wave number. Given the speed $_{+}$, the soliton amplitude $_{+}$ is implicitly determined from the soliton dispersion relation $_{+} = [\,^2_{+}(2\ln _{+} - 1) + 1]/(_{+} - 1)^2$ [29]. At the trailing edge, the modulated wave limits to zero amplitude, corresponding to harmonic waves propagating with the group velocity $_{-} = \omega'(_{-})$, where $\omega(\,) = 2\,\_/(1 + \,\_^2)$ is the linear dispersion relation of Eq. (1) on a background conduit area $\_$ and $^2_{-} = [\_ - 4 + \sqrt{\_(8 + \_)}]/(4\,\_)$ is the distinguished wave number determined from modulation theory [24] (see also Ref. [1]).

In Fig. 2, we compare the leading edge amplitude and speed predictions with the experiment, demonstrating followed by a soliton. Because solitons propagate with a nonlinear phase velocity larger than the linear wave phase and group velocities [29], the soliton eventually overtakes the DSW trailing edge. The soliton-DSW interaction results in a sequence of phase shifts between the soliton and the crests of the modulated wave train. The soliton emerges from the interaction with a significantly increased amplitude and decreased speed due to the smaller downstream conduit assume the merger of the two DSWs and thus obtain the leading edge speed of the merged DSW ¼ $4\sqrt{\frac{1}{2}ð\ _1\ þ\ \ _2þ - 1} - 1$ connecting the conduit area $_0$

Supplemental material for "Observations of dispersive shock waves, solitons, and their interactions in viscous fluid conduits"

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In this supplemental material, background information and additional experimental details are provided.