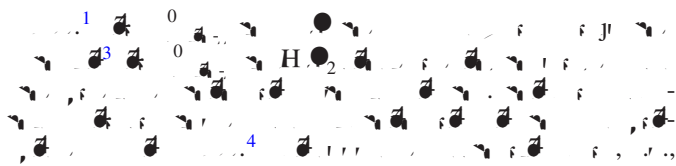


**Nonstoichiometry as a source of magnetism in otherwise nonmagnetic oxides:**

**Magnetically interacting cation vacancies and the  $T_{\text{M}}$**



$H_{21} = 10^8$

## II. METHODS

### A. Calculation of the magnetic configuration of a single vacancy

The calculation of the magnetic configuration of a single vacancy is performed using the following steps:

1. The initial configuration is defined by the spin variables  $S_i$  and the exchange interactions  $J_{ij}$ .
2. The energy of the system is calculated as a function of the spin variables.
3. The ground state configuration is determined by minimizing the energy.
4. The magnetic configuration is characterized by the spin values at the lattice sites.

The energy of the system is given by:

$$E = \sum_{\langle ij \rangle} J_{ij} S_i S_j - H \sum_i S_i$$

where  $\langle ij \rangle$  denotes nearest neighbor sites,  $J_{ij}$  is the exchange interaction, and  $H$  is the external magnetic field.

The ground state configuration is found by minimizing the energy with respect to the spin variables. This is done by solving the following equations:

$$\frac{\partial E}{\partial S_i} = 0$$

The resulting magnetic configuration is shown in Figure 1.

### C. Calculation of the magnetic interaction range

$$\Delta E_{\mathbf{M}}(\mathbf{r}) = E_{\mathbf{M}}(\mathbf{r}) - E_{\mathbf{M}}(\mathbf{0})$$

### D. Calculation of the percolation staircase

$$x = \lfloor \lambda \rfloor / N$$

$$x, (\lambda,)$$

$$x, (\lambda,)$$

1( ) 1( ),  $\dots$

**B. Results for the formation enthalpies and transition energies of Hf and O vacancies and equilibrium concentration of magnetic defects in HfO<sub>2</sub>**

3  $\dots$





$R$  (34)  
 $H$  (5, 15) (6, 41, 42)  
 $x^\lambda$   
 $(\dots)$  (43, 44)  
 $x$   
 $\mathbb{M}$   
 $M$   
 $(\dots)x^\lambda$  (45)  
 $R$  (20)  
 $R$  (33)  
 $R$  (6, 41, 42)  
 $x^\lambda$   
 $x^\lambda$   
 $\mathbb{M}$   
 $R$   
 $= R_1, R_2, R$



